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9231/12

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

- 1 (a) Give full details of the geometrical transformation in the x - y plane represented by the matrix $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [1]

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Let $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$.

- (b) The triangle DEF in the x - y plane is transformed by $\mathbf{A}$ onto triangle PQR .

Given that the area of triangle DEF is 13 cm^2 , find the area of triangle PQR . [2]

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- (c) Find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [2]

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- (d) Show that the origin is the only invariant point of the transformation in the x - y plane represented by $\mathbf{A}$. [4]

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2 It is given that $y = xe^{ax}$, where a is a constant.

Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n y}{dx^n} = (a^n x + n a^{n-1}) e^{ax}. \quad [6]$$

[illegible]

3 Let $S_n = \sum_{r=1}^n \ln \frac{r(r+2)}{(r+1)^2}$.

(a) Using the method of differences, or otherwise, show that $S_n = \ln \frac{n+2}{2(n+1)}$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

$$\text{Let } S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}.$$

(b) Find the least value of n such that $S_n - S < 0.01$.

[3]

[illegible]

4 The cubic equation $x^3 + 2x^2 + 3x + 3 = 0$ has roots α , β , γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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(b) Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$. [2]

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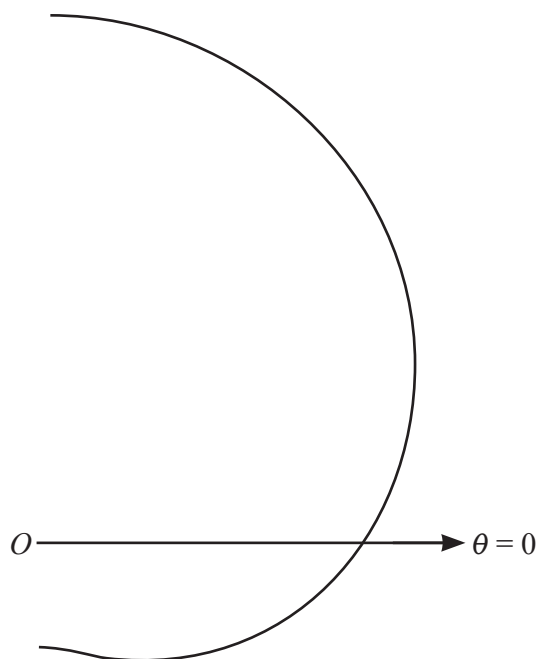
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- 5** The curve C has polar equation $r = 3 + 2 \sin \theta$, for $-\pi < \theta \leq \pi$.

- (a)** The diagram shows part of C . Sketch the rest of C on the diagram.

[1]



The straight line l has polar equation $r \sin \theta = 2$.

- (b)** Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of C and l . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- (c) The region R is enclosed by C and l , and contains the pole.

Find the area of R , giving your answer in exact form.

[6]

[illegible]

6 The curve C has equation $y = \frac{x^2}{x-3}$.

(a) Find the equations of the asymptotes of C . [3]

(b) Show that there is no point on C for which $0 < y < 12$. [4]

[illegible]

(c) Sketch C .

[2]

- (d) (i) Sketch the graphs of $y = \left| \frac{x^2}{x-3} \right|$ and $y = |x| - 3$ on a single diagram, stating the coordinates of the intersections with the axes. [4]

- (ii) Use your sketch to find the set of values of c for which $\left| \frac{x^2}{x-3} \right| \leq |x| + c$ has no solution. [1]

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7 The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j}, \quad -\mathbf{j} + \mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$$

respectively, relative to the origin O .

(a) Find an equation of the plane OAB , giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

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The plane Π has equation $x - 3y - 2z = 1$.

(b) Find the perpendicular distance of Π from the origin. [1]

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- (c) Find the acute angle between the planes OAB and Π .

[3]

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- (d) Find an equation for the common perpendicular to the lines OC and AB .

[10]

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This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for practicing various letters and words. There is no text or other markings on the page.

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